DISSERTATIO ASTRONOMICA

de

ORBITA E TRIBUS RADIIS VECTORIBUS ET ANGULIS COMPREHENSIS INVENIENDA,

Quam,

Consentiente Ampl. Philos. Ordine Lundensi,

PRÆSIDE

JONA BRAG,

Astr. & Ph. Prof. Reg. & Ord.

Pro summis in Philosophia capessendis honoribus,

exhibet

FREDRICUS E. BORG,

Scanus.

In Academia Carolina Die XII Junii MDCCCXXXII.

P. II.

LONDINI GOTHORUM,

Typis BERLINGIANIS,

MDCCCXXXII.

$$a = \frac{p}{q^2 - 1} \text{ et}$$

$$e = aq = \frac{qp}{q^2 - 1}.$$

Si $q^2 = 1$, erit $a = \frac{p}{o} = \infty$, seu parabolicus.

Si $q^2 = 1$, erit etiam

$$1 = \frac{p - r}{r \cos v}, \text{ sive}$$

$r \cos v = p - r$, ut in parabola.

Valet igitur formula p, et cum illa tangentis v, etiam pro via parabolica.

III.

Si anomalia v primum quærenda, habebimus, ut antea,

$$q = \frac{p - r}{r \cos v} = \frac{p - r'}{r' \cos (v + m)} = \frac{p - r''}{r'' \cos (v + s)}.$$

Erit igitur

$$\frac{p - r}{r \cos v} = \frac{p - r'}{r' \cos (v + m)}; \text{ sive}$$

$pr' \cos (v + m) - pr \cos v = rr' \cos (v + m) - rr' \cos v$, et

$$p = \frac{rr' (\cos (v + m) - \cos v)}{r' \cos (v + m) - r \cos v}$$

Eodem obtinebitur modo

$$p = \frac{rr'' (\cos (v + s) - \cos v)}{r'' \cos (v + s) - r \cos v}$$

Hinc

$$\frac{r' (\cos (v + m) - \cos v)}{r' \cos (v + m) - r \cos v} = \frac{r'' (\cos (v + s) - \cos v)}{r'' \cos (v + s) - r \cos v}$$

Sublatis divisoribus prodibit

$r'r'' \cos(v+m)\cos(v+s) - r'r''\cos v \cos(v+s) - rr'\cos v\cos(v+m) + rr'\cos v^2 = r'r''\cos(v+m)\cos(v+s) - r'r''\cos v\cos(v+m) - rr''\cos v\cos(v+s) + rr''\cos v^2$

Facta subtractione, divisione per cos v et reductione ad zero, erit

$r''(r-r')\cos(v+s) + r'(r''-r)\cos(v+m) + r(r'-r'')\cos v = 0$

sive, brevitatis gratia,

$A\cos(v+s) + B\cos(v+m) + C\cos v = 0$

Habebimus igitur

$A(\cos v\cos s - \sin v\sin s) + B(\cos v\cos m - \sin v\sin m) + C\cos v = 0$, sive

$A(\cos s - \operatorname{tang} v\sin s) + B(\cos m - \operatorname{tang} v\sin m) + C = 0$

Unde

$$\operatorname{tang} v = \frac{A\cos s + B\cos m + C}{A\sin s + B\sin m}$$

$$= \frac{r''(r-r')\cos s + r'(r''-r)\cos m + r(r'-r'')}{r''(r-r')\sin s + r'(r''-r)\sin m}$$

Formulam hanc, aditu facilem, et fere in promptu, neque apud Cl. Delambre, neque apud alios auctores adhuc vidimus

Concinnior fiet tollendo tertio numeratoris termino. Scilicet

$$r''(r-r')\cos s + r'(r''-r)\cos m + r(r'-r'') = r''(r-r')\left(1 - 2\sin\frac{s}{2}^2\right) + r'(r''-r)\left(1 - 2\sin\frac{m}{2}^2\right) +$$

$$r(r'-r'') = 2\left(r''(r'-r)\sin\frac{s}{2}^2 - r'(r''-r)\sin\frac{m}{2}^2\right)$$

unde

$$\operatorname{tang} v = \frac{2\left(r''(r'-r)\sin\frac{s}{2}^2 - r'(r''-r)\sin\frac{m}{2}^2\right)}{-r''(r'-r)\sin s + r'(r''-r)\sin m}, \quad (\gamma)$$

Substitutis, brevitatis gratia, $A' = -A = r''(r'-r)$, et $B = r'(r''-r)$,
habebimus porro

$$\operatorname{tang} v = \frac{A'\sin\frac{s}{2}^2 + B\sin\frac{m}{2}^2}{-A'\sin\frac{s}{2}\cos\frac{s}{2} + B\sin\frac{m}{2}\cos\frac{m}{2}}, \quad (\delta)$$

sive

$$\operatorname{tang} v = \frac{1 - \dfrac{B\sin\frac{1}{2}m^2}{A'\sin\frac{1}{2}s^2}}{-\cot\frac{s}{2} + \left(\dfrac{B\sin\frac{1}{2}m^2}{A'\sin\frac{1}{2}s^2}\right)\cot\frac{m}{2}}$$

$$= \frac{1-Q}{-\cot\frac{s}{2} + Q\cot\frac{m}{2}}, \text{ posito } \frac{B\sin\frac{1}{2}m^2}{A'\sin\frac{1}{2}s^2} = Q.$$

Quæ formula ad subducendos calculos expeditior profecto illa, ex qua Cl. Delambre (pag. 134 lib. & t. cit.) computavit $\Pi = l - v$, denotante l longitudinem heliocentricam vectori r debitam.

Ne temere dictum esse videatur, atque ut illustretur ipsa exemplo, sumamus eosdem valores vectorum r, r', r'' et angulorum m, n, s, quos, pag. 96 ib. allatos, Cl. Delambre e tabulis Martis depromsit.
$r = 1{,}392178$; $r' = 1{,}436789$; $r'' = 1{,}499689$; $m = (L' - L) = 32° \, 28' \, 16$; $n = (L'' - L') = 29° \, 0' \, 4''$, adeoque $s = m + n = 61° \, 28' \, 20''$.

$$\log r' = 0{,}1573930$$
$$\log (r'' - r) = 9{,}0314530 \quad \widehat{1}$$
$$C \log r'' = 9{,}8230990$$
$$C \log (r' - r) = 1{,}3505580 \quad \widehat{2}$$
$$2 \log \sin \frac{m}{2} = 8{,}8930338 \quad \widehat{3}$$
$$C.2 \log \sin \frac{s}{2} = 0{,}5830142 \quad \widehat{4}$$
$$\log Q = 9{,}8394510 - 10 = \log 0{,}690957 \quad \widehat{5}$$
$$\widehat{7} \log \cot \frac{m}{2} = 10{,}5358088 \qquad \log (1 - Q) = -1{,}4900189 \quad \widehat{6}$$
$$\log Q \cot \frac{m}{2} = 10{,}3752598 - 10 = \log 2{,}3727929 \quad \widehat{8}$$
$$\widehat{9} \log \cot \frac{s}{2} = 10{,}2257198 - 10 = \log 1{,}6817777 \quad \widehat{10}$$
$$\log 0{,}6910152 = -1{,}8394876 \quad \widehat{11}$$
$$\widehat{12} \log \operatorname{tang} v = \log \operatorname{tang} 24^\circ\, 5'\, 43'', 26 = -1{,}6505313 + 10$$

Si logarithmos vectorum, in tabulis planetæ cum ipsis occurrentes, quemadmodum fecit Cl. Delambre, in summam non coëgeris, tabulas logarithmicas *quattuordecies* ab illust. Auctore, *duodecies* tantum a nobis aditas invenies.

Cum in hoc exemplo sit $l = 11^s\, 26^\circ\, 29'\, 40''$ sequitur esse $\Pi = l - v = 11^s\, 26^\circ\, 29'\, 40'' - 24^\circ\, 5'\, 43'', 26 = 11^s\, 2^\circ\, 23'\, 56'', 74$. Delambre invenit $\Pi = 11^s\, 2^\circ\, 23'\, 56''$.

Verus valor ang Π, seu e Tabulis, $= 11^s\, 2^\circ\, 24'\, 14''$

error $= 18''$,

sive ex nostra computatione $= 17'', 26$ 5)

Si formulam γ adhibere placuerit, tabulæ logarithmicæ *quattuordecies* consulentur.

Ad v immediate, seu nulli cognitorum signis affectum, inveniendum, nostra via breviorem haud vidimus. Alterius vero, seu affecti generis, nonnullæ dantur paullo expeditiores.

5) De hoc errore Delambre. "*La formule a donc toute l'exactitude, qu'on peut desirer, car* 18'' *sur* Π *ne peuvent jamais produire, que* 3'' *sur la longitude.* L. et T. c. pag. 135.

IV.

Utamur ad has quærendas jam inventa formula δ, sive

$$\tang v = \frac{A' \sin\frac{s}{2}^2 - B\sin\frac{m}{2}^2}{-A'\sin\frac{s}{2}\cos\frac{s}{2} + B\sin\frac{m}{2}\cos\frac{m}{2}}.$$

Quia igitur

$$\tang\left(v + \frac{s}{2}\right) = \frac{\tang v + \tang\frac{s}{2}}{1 - \tang v \tang\frac{s}{2}}, \text{ erit}$$

$$\tang\left(v + \frac{s}{2}\right) = \frac{\dfrac{A'\sin\frac{1}{2}s^2 - B\sin\frac{1}{2}m^2}{-A'\sin\frac{1}{2}s\cos\frac{1}{2}s + B\sin\frac{1}{2}m\cos\frac{1}{2}m} + \tang\frac{1}{2}s}{1 - \left(\dfrac{A'\sin\frac{1}{2}s^2 - B\sin\frac{1}{2}m^2}{-A'\sin\frac{1}{2}s\cos\frac{1}{2}s + B\sin\frac{1}{2}m\cos\frac{1}{2}m}\right)\tang\frac{1}{2}s}$$

$$= \frac{-B\sin\frac{m}{2}^2 + B\sin\frac{m}{2}\cos\frac{m}{2}\tang\frac{s}{2}}{-A'\sin\frac{s}{2}\cos\frac{s}{2} + B\sin\frac{m}{2}\cos\frac{m}{2} - \left(A'\sin\frac{s}{2}^2 - B\sin\frac{m}{2}^2\right)\tang\frac{s}{2}}$$

$$= \frac{B\sin\frac{m}{2}\cot\frac{s}{2} + B\sin\frac{m}{2}\cos\frac{m}{2}}{+B\sin\frac{m}{2}\cos\frac{m}{2}\cot\frac{s}{2} + B\sin\frac{m}{2}^2 - A'}, \text{ divisione}$$

facta per $\tang\frac{s}{2}$.

Hinc,

$$\cot\left(v+\frac{s}{2}\right)=\frac{\frac{B}{A'}\sin\frac{m}{2}^{2}\left(\cot\frac{m}{2}\cot\frac{s}{2}+1\right)-1}{\frac{B}{A'}\sin\frac{m}{2}^{2}\left(\cot\frac{m}{2}-\cot\frac{s}{2}\right)}$$

$$=\frac{\cot\frac{m}{2}\cot\frac{s}{2}+1}{\cot\frac{m}{2}-\cot\frac{s}{2}}-\frac{1}{\frac{B}{A'}\sin\frac{m}{2}^{2}\left(\cot\frac{m}{2}-\cot\frac{s}{2}\right)}$$

$$=\cot\left(\frac{s}{2}-\frac{m}{2}\right)-\frac{1}{\frac{B}{A'}\sin\frac{m^{2}}{2}\times\frac{\sin\left(\frac{1}{2}s-\frac{1}{2}m\right)}{\sin\frac{1}{2}s\sin\frac{1}{2}m}}$$

$$=\cot\frac{n}{2}-\frac{\frac{A'}{B}\sin\frac{1}{2}s}{\sin\frac{m}{2}\sin\frac{n}{2}} \qquad (\varepsilon)$$

$$=\cot\frac{n}{2}-\frac{A'}{B}\left(\cot\frac{n}{2}+\cot\frac{m}{2}\right);\ \text{sive}$$

$$\cot\left(v+\frac{s}{2}\right)=\left(1-\frac{A'}{B}\right)\cot\frac{n}{2}-\frac{A'}{B}\cot\frac{m}{2} \qquad (\zeta)$$

$$=\left(\frac{B-A'}{B}\right)\cot\frac{n}{2}-\frac{A'}{B}\cot\frac{m}{2} \qquad (\eta)$$

Formulæ ε, ζ & η exstant omnes apud Cl. Delambre Tom. 2. pag. 135 & 136, demonstrationibus tamen, ut nobis videtur, haud satis perspicue indicatis.

Formulam, seu figuram formulæ $\cot\left(v+\frac{s}{2}\right)$ ultimam, (η), laudat Cl. Delambre, tamquam uno pauciores, quam ceteras, logarithmos exigentem. Recte, si logarithmi vectorum cum ipsis adsint. Si vero periheli-

um novæ orbitæ, nondum tabularum vinculis subjectæ, e vectoribus et angulis comprehensis, alia ratione cognitis, quæreretur, omnes illæ species, formulæ re vera ejusdem, parem logarithmorum numerum poscerent.

Ut hoc clarius pateat, inferantur valores, quorum vices supplere A' & B. Habebimus nempe,

$$\text{tang}\left(v+\frac{s}{2}\right)=\cot\frac{n}{2}-\frac{r''(r'-r)\sin\frac{1}{2}s}{r'(r''-r')\sin\frac{1}{2}m\sin\frac{1}{2}n} \qquad (\varepsilon)$$

$$\text{tang}\left(v+\frac{s}{2}\right)=\left(1-\frac{r''(r'-r)}{r'(r''-r)}\right)\cot\frac{n}{2}-\frac{r''(r'-r)}{r'(r''-r)}\cot\frac{m}{2} \qquad (\zeta)$$

$$\text{tang}\left(v+\frac{s}{2}\right)=\frac{r(r''-r')}{r'(r''-r)}\cot\frac{n}{2}-\frac{r''(r'-r)}{r'(r''-r)}\cot\frac{m}{2} \qquad (\eta)$$

Unde quatenus hoc vere dictum sit, a Cl. Delambre, satis elucet.

Computavit Ill. Auctor angulum $\Pi=(l-v)$ ex omnibus tribus figuris formulæ tangentis $\left(v+\frac{s}{2}\right)$. Ut fiat comparatio hujus methodi indirectæ, seu affectæ, cum directa illa, cujus jam formulam, eodem exemplo illustratam, dedimus, nos eligemus tantum secundam ζ.

$$\begin{aligned}
\log r'' &= 0{,}1760010\\
\log (r'-r) &= 8{,}6494420\\
C\log r' &= 9{,}8426070\\
C\log (r''-r) &= \underline{0{,}9685470}\\
\log\left(\frac{A'}{B}\right) &= 19{,}6365970 - 10 = \text{l. } 0{,}4331087\\
\log\cot\frac{m}{2} &= \underline{10{,}5358088}, \qquad 0{,}5668913 = 1-\frac{A'}{B}\\
& 10{,}1724058 - 10 = \text{l. } 1{,}487325\ (-)\\
\log\left(1-\frac{A'}{B}\right) &= {}^{-}1{,}5873246\\
\log\cot\frac{n}{2} &= \underline{10{,}7534998}\\
& 10{,}3408244 - 10 = \text{l. } \underline{2{,}191919}\ (+)\\
\log\cot(54^\circ\,49'\,54'') &= 10+\log 0{,}704594 = 9{,}8479390.
\end{aligned}$$

$v = 54° 49' 54'' - 30° 44' 10'' = 24° 5' 44''$

Requirit igitur hæc methodus ad tabulas logarithmicas aditus, duobus pauciores, quam directa illa, seu non affecta, per tang v pag. 11.

Formula Gaussiana (pag. 89 l. c.) ita data tang $(\frac{1}{2} N + \frac{1}{2} N'' - \Pi)$

$$= \frac{\frac{r'}{r} - \frac{r'}{r''}}{\left(1 - \frac{r'}{r''}\right) \cot \frac{1}{2} (N'' - N') - \left(\frac{r'}{r} - 1\right) \cot \frac{1}{2} (N' - N)}$$, nova species est formulæ $\cot\left(v + \frac{s}{2}\right) = \left(1 - \frac{A'}{B}\right) \cot \frac{n}{2} - \frac{A'}{B} \cot \frac{m}{2}$, ad cujus figuram facile revocabitur. Quomodo vero ipsa, et aliæ, hujus propositi, angulisque sic dictis auxiliaribus innixæ formulæ, suis inveniantur methodis, vir Cl. ibidem ingeniose ostendit.

Ut tollatur signi ambiguitas, seu decernatur, in quo quadrantum quattuor situs sit v, quærendus est primus semiparameter, positivæ semper naturæ, nec ulla ratione a varietate illa signorum dependens.

Est autem $p = \frac{rr'' (\cos (v + m) - \cos v)}{r' \cos (v + m) - r \cos v}$, ubi utrumvis signorum cosinui ang. v, vel cos. ang $(v + m)$ datum fuerit, collecti numeratoris et denominatoris valores vel simul positivi erunt, vel simul negativi.

Invento p et cum r comparato, e formula $q = \frac{p - r}{r \cos v}$ statim in notescet signum cosinus v.

Nondum tamen satis liquet, si positivus cos v, utrum in 1:mo an 4:to, si negativus, utrum in 2:do, an 3:tio quadrante constituendus.

Etenim, si r & r' in eadem fuerint semiellipsi, quod usu fere venit, valebit omnino regula, quam, hoc respicientes, pag. 6. part. 1. dedimus. Cum vero, geometria saltem non prohibente, etiam in diversa jacere possint, quoties $r > r'$, manebit adhuc loci ambiguitas. Ut tollatur penitus, repetamus formulam tangentis $v = \frac{\cos m - M}{\sin m} = \frac{\cos m - \frac{r (p - r')}{r' (p - r)}}{\sin m}$, ubi, si $r > r'$, sed $p > r$, M per se positivus erit et unitate major; si $r > p$, sed $p > r'$, M negativus et unitate vel major vel minor; si denique $r > r'$ sed $r' > p$, M positivus & unitate minor. In primo igitur casu signum tangentis e signo sinus m, inverso ordine, pendebit. In secundo, si sinus m fuerit negativus, ex ipsa constructione datorum statim patebit tang v esse negativum, adeoque hic necessario $M >$ cos m. Si vero *sin* m positivus fuerit et cos m negativus, erit tang v, vel neg. vel pos., prout cos m vel $>$ vel $<$ M. Si fuerit cos m positivus, erit etiam tang v. In ultimo denique, ubi sin m semper pos., si cos m negativus fuerit, erit etiam tang v; sin vero cos m

positivus, erit tang v, vel neg., vel pos., prout fuerit cos m vel $<$ vel $> M$

Valebunt formulæ anguli v, primum inquirendi, etiam pro Hyperbola. Limitantur tamen natura curvæ.

Si enim fuerit $\cos v = -\frac{1}{q} = -\frac{a}{e}$, erit $r = \frac{p}{q \cos v + 1} = \frac{p}{0} = \infty$.

In Parabola, ubi $q = 1$, duobus tantum vectoribus cum angulo comprehenso opus est ad v primum omnium inveniendum.
Erit enim

$$r\ (1 + \cos v) = r' \left(1 + \cos (v + m)\right),\ \text{sive}\ r \cos \frac{v}{2}^2 = r' \cos \left(\frac{v + m}{2}\right)^2$$

unde

$$\sqrt{\frac{r}{r'}} = \cos \frac{m}{2} - \operatorname{tang} v \sin \frac{m}{2},\ \text{sive}$$

$$\operatorname{tang} v = \frac{\cos \frac{m}{2} - \sqrt{\frac{r}{r'}}}{\sin \frac{m}{2}}$$

Si $\cos v = \cos 180° = -1$, erit $r = \frac{p}{\cos (v + 1)} = \frac{p}{0} = \infty$.

Errores observati.

P. I pag. 1 linea 8. Deest (.) post parenthesin
— — 8 linea 8. $r \cos v'$ pro $r \cos v$